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ANALYSIS OF THE HYDRODYNAMIC INTERACTION BETWEEN CASCADES OF THIN PROFILES TAKING ACCOUNT OF VORTEX WAKE EVOLUTION

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UDC 532.582.2

The papers [1-5] are devoted to an investigation of aspects of the hydrodynamic interaction of cascades of profiles in a nonlinear formulation: it is shown experimentally in [1] and theoretically in [2] that the free vortex sheet ruptures upon meeting a profile; taking account of the evolution of vortex wakes, the flows around two cascades of solid profiles of infinitesimal [3] and finite [4] density are computed; results of an experimental investigation of the dynamic reactions of the flow on two mutually moving cascades of thin profiles are presented in [5]. The interference between two cascades of thin profiles in an inviscid, incompressible fluid flow is examined in this paper, where a modified method from [6] is used.

§1. Undetached flow around two cascades of thin profiles by an inviscid incompressible fluid is considered in the plane of the x, y Cartesian coordinates. The y axis is directed along the front of the cascades. The left cascade is assumed fixed, while the right cascade moves along the y axis at the velocity u = const. The flow outside the profiles and their shed vortex wakes are assumed potential, the cascade spacings are identical, the profiles are rigid, and the influence of the wake and profile thicknesses is negligible.

Under the assumptions made, the flow velocity $\mathbf{V} = (V_x, V_y)$ satisfies the equations

$$\operatorname{div} \mathbf{V} = 0, \ \operatorname{rot} \mathbf{V} = 0, \ (x, y) \notin L; \tag{1.1}$$

the periodicity condition

$$\mathbf{V}(x, y + h, t) = \mathbf{V}(x, y, t) \tag{1.2}$$

and the following boundary conditions:

nonpenetration of the fluid through the profile of the cascades

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 61-65, July-August, 1976. Original article submitted July 8, 1975.

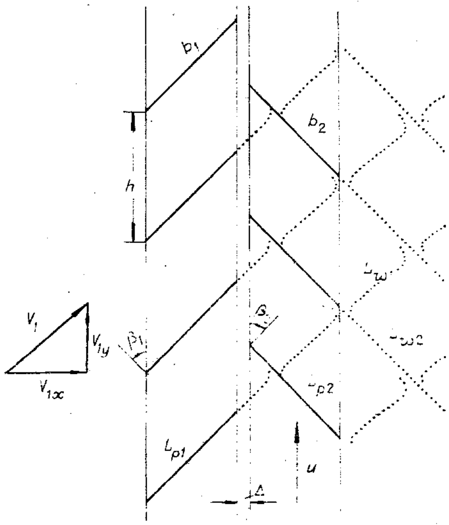

Fig. 1

$$\mathbf{V}\cdot\boldsymbol{\nu} = \delta_{k2}\mathbf{u}\cdot\boldsymbol{\nu},\ (x,\ y) \in L_{pk},\ k = 1,\ 2; \tag{1.3}$$

nonpenetration of the fluid through the vortex wakes

$$\mathbf{V}\cdot\boldsymbol{\nu} = \mathbf{V}_w\cdot\boldsymbol{\nu},\ (x,\ y) \in L_w; \tag{1.4}$$

continuity of the pressure during passage through the wake

$$[p] = 0,\ (x,\ y) \in L_w; \tag{1.5}$$

continuity of the pressure at the trailing edges of the profiles (Kutta–Zhukovskii condition)

$$[p] \to 0,\ (x,\ y) \to (x_e,\ y_e),\ (x,\ y) \in L_p,\ (x_e,\ y_e) \in E_p; \tag{1.6}$$

damping of the perturbed flow velocity infinitely far in front of the cascades

$$\lim_{x\to-\infty} \mathbf{V}(x,\ y,\ t) = \mathbf{V}_1. \tag{1.7}$$

Here t is the time; h is the cascade spacing; L_{p1} and L_{p2} are the contours of the fixed and mobile cascades, respectively (Fig. 1); $L_p = L_{p1} + L_{p2}$; $L_w = L_{w1} + L_{w2}$; L_{w1} and L_{w2} are the contours of the vortex wakes shed from L_{p1} and L_{p2}, respectively; $L = L_p + L_w$; $\boldsymbol{\nu}$ is the unit normal to L; $\mathbf{u} = (0,\ u)$; δ_{k2} is the Kronecker delta; $\mathbf{V}_w$ is the displacement velocity of the line L_w; p is the hydrodynamic pressure; E_p is the set of profile trailing edges; and $\mathbf{V}_1 = (V_{1x},\ V_{1y}) = \text{const}$.

At the initial instant it is assumed that there are no vortex wakes and the cascade configuration is given:

$$L|_{t=0} = L_p|_{t=0} = L_{p0}. \tag{1.8}$$

The problem (1.1)–(1.8) is nonlinear, since the contour $L_w(t)$ is not known in advance.

§2. Let L^0_{pk} be some profile of the k-th cascade, selected as the initial cascade; let L^0_{wk} be the contour of the wake shed from L^0_{pk} (k = 1, 2). Let us introduce the quantity $\mathbf{v} = (v_x,\ v_y)$ by means of the equality

$$\bar{v}(s,\ t) = \overline{V}_1 + \frac{1}{2hi}\int_{L^0} \gamma(\sigma,\ t)\left\{1 + \operatorname{cth}\frac{\pi}{h}[\zeta(s,\ t) - \zeta(\sigma,\ t)]\right\}d\sigma, \tag{2.1}$$

where $\bar{v} = v_x - iv_y$; $\overline{V}_1 = V_{1x} - iV_{1y}$; s, σ is the arc abscissa, $L^0 = L^0_{p1} + L^0_{p2} + L^0_{w1} + L^0_{w2}$; γ is the vortex layer intensity, $\zeta = \xi + i\eta$ is a complex coordinate of a point of the contour L^0, and the integral is understood in the Cauchy principal value sense. Taking account of (2.1), the problem (1.1)–(1.8) reduces, by analogy with [6], to a problem in the function γ on the contour L^0 and in the function ζ on the contour $L^0_w = L^0_{w1} + L^0_{w2}$. It is hence assumed that each point of an arbitrary element $(\sigma'(t), \sigma''(t))$ of the contour L^0_w moves at the velocity **v**, and the total vortex intensity on the element remains unchanged:

$$\int\limits_{\sigma'(t)}^{\sigma''(t)} \gamma(\sigma, t)\, d\sigma = \text{const.} \tag{2.2}$$

Then the intensity of the free vortices shed from the contour $L^0_p = L^0_{p1} + L^0_{p2}$ is expressed by the formula

$$\gamma(l, t) = -\frac{1}{w(l,t)} \frac{d}{dt} \int\limits_0^l \gamma(\sigma, t)\, d\sigma, \tag{2.3}$$

where l is the length of the contour L^0_{pk}; σ is measured from the profile leading edge; w is the relative velocity of free vortices being shed from the profile determined by the equality

$$w(s, t) = \operatorname{Re}\{\bar{v}\partial\zeta/\partial s\} - \delta_{k2}u\partial\eta/\partial s, \ k = 1, 2. \tag{2.4}$$

Taking account of (2.1), condition (1.3) becomes

$$\operatorname{Im}\{\bar{v}\partial\zeta/\partial s\} = -\delta_{k2}u\partial\xi/\partial s, \ k = 1, 2. \tag{2.5}$$

The initial condition can be written as

$$L^0|_{t=0} = L^0_p|_{t=0} = L^0_{p0}. \tag{2.6}$$

Let us note that compliance with the boundary conditions (1.3), (1.4) and (1.5), (1.6) follows, respectively, from (2.5), (2.1) and (2.2), (2.3), and (2.4), but (1.1) and the conditions (1.2), (1.7) are automatically satisfied because of replacement of the contour L^0 by a vortex layer. Starting from this, let us henceforth consider the problem (2.1)-(2.6) of motion of the vortex layer L^0.

§3. The algorithm in [6] is used for the numerical solution of the problem (2.1)-(2.6). This algorithm is based on linearizing the problem in a small neighborhood of each time and approximating the vortex layer by a system of discrete vortices. In addition, the following factors are taken into account in the algorithm.

Selection of the Time Spacing Δt. The spacing Δt in [6] satisfies the condition

$$\Delta t = l/(wN), \tag{3.1}$$

where N is the number of vortices per profile. Condition (3.1) assures the uniformity of the vortex distribution in the neighborhood of the profile trailing edge. In the problem under consideration the spacing Δt is given as constant beforehand, since condition (3.1) cannot be satisfied simultaneously in both cascades. The selection of the constant Δt results in some error in the determination of the vortex intensity on the profile and its shed free vortex.

A numerical investigation of the factors on which the magnitude of these errors depends would result in the following. In the neighborhood of the profile trailing edge the relative error ε_γ in determining the vortex intensity depends for $N \geq 20$ on just the dimensionless parameter $\tau = l/(wN\Delta t)$ and the vortex number, in practice. This error should not be neglected if $\tau \neq 1$. For example, for $\tau = 1/4; 1/2; 1; 2; 6$ the quantity ε_γ for a free vortex has the respective values −0.63; −0.28; 0.00; 0.18; 0.36. In conformity with this, the vortex intensity at each time is calculated formally by the method in [6] at the beginning by ignoring condition (3.1). Then the parameter τ and its corresponding quantity ε_γ are determined and the errors are eliminated.

Passage of the Second Cascade L_{p2} through the Wake L_{w1} from the First Cascade. It has been shown experimentally in [1] and theoretically in [2] that the ends of the parts of the vortex wake slit by the profile diverge. Then the intensity of the free vortices at contiguous points of the wake and the profile should be zero (otherwise, the velocity of the motion of these points turns out to be infinite, which has no physical meaning). Hence, sections of the wake L_{w1} located in a small neighborhood of the cascade profile L_{p2} can be neglected because of the low intensity of the free vortices in these sections. Now, the method in [6] can be used to solve the problem by taking into account that the system of discrete vortices yields a satisfactory approximation to the velocity field of the vortex layer only at a sufficient distance away. In conformity with this, the discrete vortices modeling the wake L_{w1} are not permitted to approach the profiles of the cascade L_{p2} to a distance less than

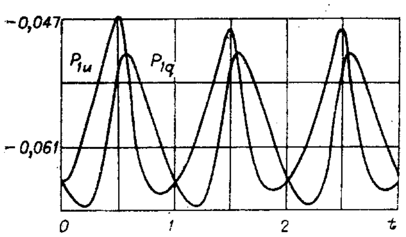

Fig. 2

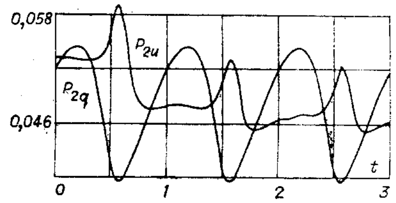

Fig. 3

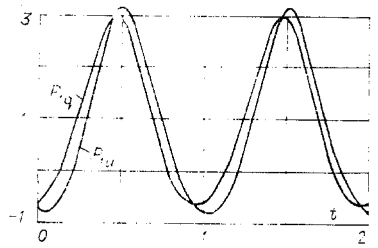

Fig. 4

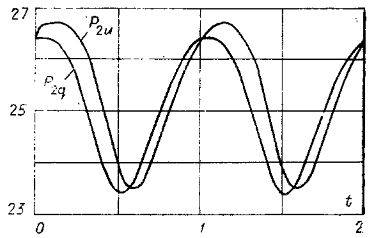

Fig. 5

$\varkappa l/N$. The parameter $\varkappa$ is selected from a numerical experiment in such a way that the hydrodynamic reactions of the stream on the profile L^0_{p2} were calculated with minimum error (usually $1 \leq \varkappa \leq 2$). The errors mentioned are minimal if the results of a duplicate computation of the problem with twice the number of vortices on the profiles of both cascades and with half the time spacing are similar to those obtained initially. Failure of the procedure described above (which corresponds to the value $\varkappa = 0$) ordinarily results in passage of the free vortices through the profile L^0_{p2}.

Results of a computation of the interference between cascades are presented for two cases as an illustration. Common to both is the fact that the contours L^0_{p1} and L^0_{p2} are rectilinear, and the profile chords coincide with the cascade spacing $b_1 = b_2 = h$ (Fig. 1). The problem is considered in both the nonstationary formulation (2.1)-(2.6) and in the quasistationary formulation (without taking account of vortex wakes). The quantities P_{uk} and P_{qk} denote the nonstationary and quasistationary resultant normal pressure forces of the stream on the profile L^0_{pk}, respectively; N_k is the number of discrete vortices modeling L^0_{pk} ($k = 1, 2$). The quantities h/u and ρhu^2, respectively, are taken as the unit of time and force (ρ is the fluid density).

1. The cascade stagger angles are $\beta_1 = \pi/4$, $\beta_2 = -\pi/4$, the gap between the cascades is $\Delta = 0.1$ h, the velocity is $V_1 = (0.7u \cos 40°, 0.7u \sin 40°)$, the time spacing is $\Delta t = 1/32$, and the parameters are $N_1 = 20$, $N_2 = 40$, $\varkappa = 1$. The computed shape of the vortex wakes presented in Fig. 1 for the time $t = 3$ indicates that the wake of the first cascade L_{w1} experiences tension and strain near the profiles of the second cascade, while the wake L_{w2} is practically without strain. The ends of the slit parts of the wake L_{w1} diverge. The described singularities of the vortex wake motion agree with [1-4].

The time dependence of the nonstationary P_{uk} and quasistationary P_{qk} forces is presented in Figs. 2 and 3. The behavior of these forces differs noticeably, especially on the second cascade. It should be noted that an analogous variant of the interference between cascades consiting of symmetric 5% thick solid profiles has been computed in [4]. It is indicated in that paper that the mean value of the quantities $|P_{u1}|$ and $|P_{u2}|$ per period is 0.07247 and 0.07154, respectively. These data do not agree satisfactorily with Figs. 2 and 3.

2. The stagger angles are $\beta_1 = 0$, $\beta_2 = -\pi/6$, the gap is $\Delta = 0.2$h, the velocity is $V_1 = (7.536u, 0.129u)$; the time spacing is $\Delta t = 1/64$, and the parameters are $N_1 = N_2 = 20$, $\varkappa = 1$. In this case, the contrast in the behavior of the nonstationary and quasistationary reactions is negligible (Figs. 4 and 5). Upon comparing Figs. 3 and 5, the importance of the dimensionless parameter u/V_{1x}, the analog of the Strouhal number in this problem, becomes clear.

The author is grateful to D. H. Gorelov for discussing the research.

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BOUNDARY LAYER ON A ROTATING CYLINDER IN AXIAL FLOW

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A semiinfinite hollow cylinder of radius R is rotating about its own axis at an angular velocity ω, and an incompressible liquid flows around it in uniform flow at a velocity u_∞. The flow is assumed to be laminar and axisymmetrical. The variables

$$s = \beta\xi;\ \ \eta = (r^2 - R^2)/2\xi R^2;$$
$$\varphi = \psi/u_\infty\xi R^2;\ \ w = w^*/\omega R;\ \ p = (p^* - p_\infty)/\rho u_\infty^2 \tag{1}$$

are used to solved the problem, where

$$\xi = \sqrt{(\nu x/u_\infty)}/R;\ \beta = (\omega R/u_\infty)^2; \tag{2}$$

ξ and η are analogous to the variables proposed in [2] for the case of a nonrotating cylinder; ψ is the stream function, which is defined by the relations $u^* = \psi_r/r$ and $v^* = -\psi_x/r$; x is the distance from the origin of the cylinder along the generating line; r is the distance to the axis of the cylinder; u^*, v^*, and w^* are the longitudinal, radial, and circumferential components of the velocity; p^* is the pressure; p_∞ is the pressure in the advancing flow; ρ is the density of the liquid; and ν is the kinematic modulus of viscosity. From here on an independent variable which appears as a subscript denotes differentiation with respect to it. The relation $u^*/u_\infty = u = \varphi_\eta$ is valid for the longitudinal velocity component.

In the dynamical equations, terms of the order $(\mathrm{Re}^x)^{-1} = \nu/u_\infty x$ are discarded, i.e., an approximation to the boundary layer is used,

$$2(\sigma\varphi_{\eta\eta})_\eta + \varphi\varphi_{\eta\eta} + \eta p_\eta - sp_s = s(\varphi_\eta\varphi_{\eta s} - \varphi_s\varphi_{\eta\eta});$$
$$2(\sigma w_\eta)_\eta + \varphi w_\eta + (\xi/\sigma)(\varphi - \eta\varphi_\eta + s\varphi_s - 2\xi)w = s(\varphi_\eta w_s - \varphi_s w_\eta),$$
$$p_\eta = sw^2/\sigma,\ \sigma = 1 + 2\xi\eta. \tag{3}$$

The flow around the exterior surface is investigated, and the boundary conditions are of the form

$$\varphi_\eta = \varphi = 0,\ w = 1 \text{ when } \eta = 0;$$
$$\varphi_\eta = 0,\ w = p = 0 \text{ as } \eta \to \infty. \tag{4}$$

The case in which the ratio of the thickness of the boundary layer to the radius of the cylinder is small, i.e., $\xi \ll 1$, is of special interest. A limiting transition is possible in Eqs. (3) as $\xi \to 0$ (or as $\beta \to \infty$)

$$2\varphi_{\eta\eta\eta} + \varphi\varphi_{\eta\eta} + \eta p_\eta - sp_s = s(\varphi_\eta\varphi_{\eta s} - \varphi_s\varphi_{\eta\eta});$$
$$2w_{\eta\eta} + \varphi w_\eta = s(\varphi_\eta w_s - \varphi_s w_\eta),\ p_\eta = sw^2. \tag{5}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 65-70, July-August, 1976. Original article submitted September 16, 1975.